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Erasmus Mundus Europhotonics
Karlsruhe School of Optics & Photonics (KSOP)
Transformation Optics: From Lenses to Metamaterials

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Outline

- The elder Transformation optics: Eaton lens
- Other kinds of Spherical GRIN lenses
- Effective medium design of a structured GRIN lens
- Photonic crystal carpet
- Homogeneous optical cloak
- Conclusion
Eaton lens

\[
\left( \nabla^2 + n^2 \frac{\omega^2}{c^2} \right) \psi = 0,
\]

Helmholtz equation

\[
E \Psi = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi
\]

Stationary Schrödinger equation

\[
U - E = -\frac{n^2}{2}.
\]

\[
n(r) = \sqrt{\varepsilon_r \mu_r} = \sqrt{\frac{2a}{r} - 1} \quad \text{for } r \leq a \quad \text{and} \quad n = 1 \quad \text{for } r > a,
\]

The Eaton lens corresponds to the Kepler potential \( U = -a/r' \) for \( r' < a \) and \( U = -1 \) outside of the device, with total energy \( E = -1/2 \). The fictitious particle draws a half Kepler ellipse around the centre of attraction. So it leaves in precisely the opposite direction it came from: light is retroreflected.
Eaton lens

Multilayer approximation (ten layers) of the Eaton lens for a $180^\circ$ bending
Spherical GRIN lenses

- **90° Eaton lens**
- **360° Eaton lens**
- **Luneburg lens**
The Mercator projection is a cylindrical map projection presented by the Flemish (Belgian) geographer and cartographer Gerardus Mercator, in 1569.
Optical Conformal Mapping

- How does the conformal transformation act on light waves?

\[
\left( \nabla^2 + \frac{\omega^2}{c^2 n^2} \right) \psi = 0
\]

Helmholtz equation

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = \left( \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right) \left( \frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right).
\]

\[
\nabla^2 = 4 \frac{\partial^2}{\partial z \partial z^*} = 4 \frac{dw}{dz} \frac{dw^*}{dz^*} \frac{\partial^2}{\partial w \partial w^*} = \left| \frac{dw}{dz} \right|^2 \nabla'^2.
\]

\[
z = x + iy, \\
x' + i y' = w(z)
\]

\[
n = \left| \frac{dw}{dz} \right| n'.
\]

Ulf Leonhardt, SCIENCE, VOL 312, 23 JUNE 2006
Transformation optics design

- we consider the line element on a sphere of radius $a$:

$$ds^2 = a^2 \sin^2 \theta \left( \frac{1}{\sin^2 \theta} d\theta^2 + d\phi^2 \right)$$

with the coordinates $(\theta, \phi)$ of the spherical polar system. Under the geometric transform (Mercator projection)

$$u = \ln(\tan(\theta/2)) \ , \ \theta = 2\tan^{-1}(e^u)$$

such that $d\theta = \sin \theta \cdot du$ and $\sin \theta = \text{sech}(u)$

$$ds^2 = a^2 \text{sech}^2 u \left( du^2 + d\phi^2 \right)$$

$$n(r) = n_0 \text{sech}(\alpha r)$$

where $\alpha = \frac{1}{R_0} \cosh^{-1} \left( \frac{n_0}{n_R} \right)$
Gradient index optics design

Fig. 1. Numerical validation at wavelength $\lambda = 700$ nm for a GRIN lens of thickness and radius $R_0$ of 1000 nm. (a) Profile of the refractive index versus radius; (b) Three-dimensional plot of the norm of electric field; (c) Side view of the real part of y-component of electric field; (d) Side view of norm of electric field.
Effective medium design of a structured GRIN lens

- We note that the effective permittivity $\varepsilon_e$ of the composite medium is given by the classical Maxwell-Garnett formula

$$\frac{\varepsilon_e - \varepsilon}{\varepsilon_e + \varepsilon} = \frac{\varepsilon_0 - \varepsilon}{\varepsilon_0 + \varepsilon} f$$

(a) Three-dimensional diagrammatic view of the structured GRIN lens; (b) Side view where $R$ denotes the radius of the lens and $r$ the radius of toroidal air channels; (c) Top view; (d) Table of filling fraction and effective index of refraction.

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<th>$r$ (nm)</th>
<th>$n_{\text{eff}}$</th>
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Illustrative numerical examples

Fig. 3. Numerical validation at wavelength $\lambda = 700$ nm for a GRIN lens of thickness and radius $R_0$ of 1000 nm. (a) The total energy density varies with optical axis ($\alpha\beta$). Dashed box indicates the location of GRIN lens; (b) Three-dimensional plot of the real part of the the y-component of electric field; (c) Side view of the y-component of electric field; (d) Side view of norm of electric field.
The regions in cyan are transformed into each other. Shaded regions represent the ground planes. The observer perceives the physical system as the virtual one with a flat ground plane.

Incident wave

Identical radius of pillar

Modified radius of pillar
Homogeneous optical cloak

\[ \rho' = \rho, \varphi' = \varphi, z' = \frac{H_2}{H_2 - H_1} \left( z - \frac{R - \rho}{R} H_1 \right), \text{ for } z > 0 \]

\[ \bar{\varepsilon} = \varepsilon_b \left[ \pm \left( \frac{H_2}{H_2 - H_1} \right)^2 \frac{H_1}{D} \right] \pm \left( \frac{H_2}{H_2 - H_1} \right)^2 \frac{H_1}{D} \right] 1 + \left( \frac{H_2}{H_2 - H_1} \right)^2 \left( \frac{H_1}{D} \right)^2 \].
Homogeneous optical cloak

TM mode
Without carpet
degree from normal incident: 30

TM mode
Without carpet
degree from normal incident: 45

TM mode
Without carpet
degree from normal incident: 60

TM mode
With carpet
Uniform layered structures

\[ \bar{\varepsilon} = \varepsilon_b \pm \frac{H_2}{H_2 - H_1} \left( \frac{H_2}{H_2 - H_1} \right)^2 \frac{H_1}{D} \left( \frac{H_1}{D} \right)^2. \]

Diagonalization

\[
\begin{bmatrix}
\varepsilon_{\parallel} & 0 \\
0 & \varepsilon_{\perp}
\end{bmatrix}
\]

The permittivity tensor can be diagonalized by rotating the optical axis by the angle \( \theta \). According to the effective medium theory, the anisotropic parameters can be realized with alternating layered materials.

degree from normal incident: 30

degree from normal incident: 45

degree from normal incident: 60
SPP carpet

Without carpet

With ideal carpet

With effective carpet
Concussion

- Investigating the tunable focusing features of a cylindrical GRIN lens and proposed a practical design of PMMA matrix drilled with toroidal air channels.

- The photonic crystal carpet has been fabricated using an original technique based on a one-mask process which makes it possible to create pillars and holes with an etching depth larger than 1.5um.

- A homogeneous invisibility cloak constructed with uniform layered structures.
Acknowledgment
Thank you for your attention
Illustrative numerical examples

Fig. 4. (a) The total energy density varies with the distance from lens along optical axis; (b) The distributions of total energy density on the focal plane $\beta$ in panel (c); (c) Side view of total energy density; (d) The distributions of total energy density from panel (c).
**Focussing effect versus wavelength**

Fig. 5. Total energy density for wavelengths of (a) 600 nm; (b) 850 nm; (c) 1000 nm; (d) Variation of total energy density versus z axis. Dashed box indicates the location of lens.
**Focussing effect versus geometric perturbation**

Fig. 6. Calculated magnitude of energy density ($J/m^3$) for a wavelength of 700 nm (a) Only PMMA; (b) Effective structure with inner air channels removed; (c) Complete structure; (d) The effective refractive index for each toroidal unit cell (dotted green line and solid red lines are the effective index in panels (b) and (c), respectively). The dashed blue curve is the ideal index from equation (1) for comparison. (e) Three-dimensional diagrammatic view of the split ring GRIN lens; (f) Three-dimensional plot of the energy density.
covariant derivative of a vector